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NUMERICAL SIMULATION AND OPTIMIZATION OF THE PROCESSES OF MICROWAVE TREATMENT OF DIELECTRICS

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The processes of electrodynamics, heat and mass transfer, and thermomechanics in a dielectric under the action of microwave energy are simulated numerically. A method for solving problems of optimization of thermal treatment of materials in microwave apparatus using beam-type chambers is proposed.

The microwave thermal treatment of dielectric materials has been used in various industries because of the high concentration of energy, rapid volumetric heating, and the possibility of obtaining the required distributions of heat sources and temperature field in the heated object [1]. Along with the conventional use of microwave energy for heating, drying, and defrosting, there has been increased interest in ultrahigh frequencies in high-temperature technologies, for example, production of refractory and heat-insulating materials, sintering and annealing of ceramics, and thermal hardening of ground blocks [2]. Many of these processes are accompanied by phase changes and high temperature stresses and deformations. Therefore, the development of adequate mathematical models for the physical processes occurring in dielectrics under the action of microwave energy is an important problem. Such models can be used to control technological processes and design optimal microwave apparatus.

Microwave apparatus with beam-type chambers make it possible to perform thermal treatment of objects with various overall dimensions, ensuring high power, uniform heating, and periodic and methodical modes of operation. Among such chambers are chambers with limited and unlimited volumes whose dimensions far exceed the wavelength [1]. Horn or slot antennas are usually employed as emitters.

We first consider processes of microwave thermal treatment that can occur in apparatus of any type. When a dielectric is placed in a microwave field, the temperature of the object increases due to dielectric losses, temperature strains and stresses arise, and mass-transfer processes can take place. We describe these processes mathematically taking into account the assumptions given below.

For a large class of materials, except for ferromagnetics and ferroelectrics, the electric properties do not depend on the electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ intensities. With allowance for this, the constitutive equations for an isotropic medium are written as

$$\boldsymbol{D} = \varepsilon \boldsymbol{E}, \qquad \boldsymbol{B} = \mu \boldsymbol{H}, \qquad j = \gamma \boldsymbol{E},$$

where $\boldsymbol{D}$ and $\boldsymbol{B}$ are the electric and magnetic induction vectors, ε and μ are the absolute dielectric and magnetic permittivities of the medium, γ is the electrical conductivity, and j is the conduction current density.

We assume that extraneous currents and bulk charges are absent, and the medium has a uniform capillary–porous structure, in which there are no chemical transformations. Following [3, 4], we assume that

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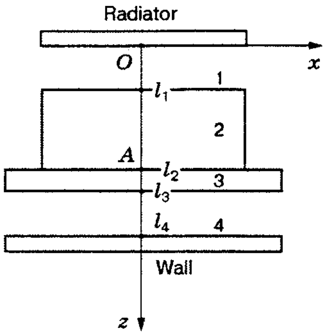

Fig. 1. Propagation of an electromagnetic wave in a multilayer medium: 1, 4) air; 2) dielectric; 3) conveyer belt.

the thermal and mechanical properties are constant and the dependence between the mechanical stresses and strains is linear in narrow ranges of temperature and time.

In this case, the processes of microwave thermal treatment of dielectrics are described by the equations

$$\operatorname{rot} \boldsymbol{H} = j + \frac{\partial \boldsymbol{D}}{\partial \tau}, \quad \operatorname{rot} \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial \tau}, \quad \operatorname{div} \boldsymbol{D} = 0, \quad \operatorname{div} \boldsymbol{B} = 0; \tag{1}$$

$$\frac{\partial T}{\partial \tau} + \boldsymbol{v} \cdot \nabla T = k_{11} \nabla^2 T + k_{12} \nabla^2 W + k_{13} \nabla^2 p + \frac{q_v}{c\rho} - \frac{(3l + 2m)\alpha T_0}{\lambda} \cdot \operatorname{div} \boldsymbol{u}; \tag{2}$$

$$\begin{gathered} \frac{\partial W}{\partial \tau} + \boldsymbol{v} \cdot \nabla W = k_{21} \nabla^2 T + k_{22} \nabla^2 W + k_{23} \nabla^2 p, \\ \frac{\partial p}{\partial \tau} + \boldsymbol{v} \cdot \nabla p = k_{31} \nabla^2 T + k_{32} \nabla^2 W + k_{33} \nabla^2 p; \end{gathered} \tag{3}$$

$$m \nabla^2 \boldsymbol{u} + (l + m) \cdot \operatorname{grad} (\operatorname{div} \boldsymbol{u}) + \boldsymbol{X} - (3l + 2m)\alpha \operatorname{grad} (T - T_0) = \rho \frac{\partial^2 \boldsymbol{u}}{\partial \tau^2}; \tag{4}$$

$$\sigma_{ij} = 2m e_{ij} + [l e_{ij} - (3l + 2m)\alpha(T - T_0)]\delta_{ij}, \quad e_{ij} = e_{ji} = 0.5(u_{i,j} + u_{j,i}). \tag{5}$$

Equations (1) are the Maxwell equations, Eqs. (2) and (3) describe heat and mass transfer, and Eqs. (4) and (5) are the equations of thermomechanics. The following designations are used: T, W, and p are functions of temperature, moisture content, and pressure, τ is time, $\boldsymbol{u}$ is the displacement vector, $\boldsymbol{v}$ is the vector velocity of motion, λ, c, and ρ are the thermal conductivity, specific heat at constant pressure, and density of the dielectric, $q_v = 0.5\omega\varepsilon_0\varepsilon' \tan \delta |E|^2$ is the power of internal heat sources due to dielectric loss in the microwave field, ω is the circular frequency, ε_0 is the absolute dielectric permittivity of vacuum, ε' is the relative permittivity of the material being treated, $\tan \delta$ is the loss tangent, k_{11}, k_{12}, ..., and k_{33} are the heat- and mass-transfer coefficients [3], $l = \nu M/[(1 + \nu)(1 - 2\nu)]$ is the Lamé coefficient, M and ν are the modulus of elasticity and Poisson's constant, $m = M/[2(1 + \nu)]$ is the shear modulus, α is the mean thermal-expansion coefficient in the temperature range $[T, T_0]$, $\boldsymbol{X}$ is the volumetric force vector, σ_{ij} are the mechanical stress tensor components (i, $j = 1, 2, 3$), e_{ij} are the strain tensor components, $u_{ij} = \partial u_i / \partial x_i$ are derivatives with respect to space coordinates, u_i are the displacement vector components in the x, y, and z directions, which are denoted by x_i ($i = 1$, 2, and 3), δ_{ij} is the Kronecker delta having values $\delta_{ij} = 0$ ($i \neq j$), and $\delta_{11} = \delta_{22} = \delta_{33} = 1$. The above equations should be supplemented by appropriate initial and boundary conditions [1, 3, 4].

As follows from Eqs. (1)–(5), the electrodynamic, thermal, and mechanical processes occurring in the dielectric under microwave thermal treatment are interrelated. For example, in microwave drying, unsteady temperature and pressure fields give rise to strains and temperature stresses. At the same time, strains lead to a change in the temperature of the object (2). In addition, strains exert an influence on the propagation of electromagnetic waves, which is explained as follows. Because of temperature strains, the spatial location of the boundaries of the dielectric changes. If the dielectric dimensions and the wavelength are commensurable, strains lead to a change in the electric intensity because the phase ratios change with interference of e.g., the incident and reflected waves.

The relationship between the processes considered is also due to the fact that the mechanical, electric, and thermal properties of dielectrics can depend on temperature and moisture content.

Equations (1)–(5) can be simplified, depending on the design of microwave apparatus and the type of technological process. We consider the methods of solving these equations as applied to the processes of thermal treatment of dielectrics in a microwave chamber of the beam type (Fig. 1). We assume that the dielectric being treated is on a conveyer belt made of a radiotranslucent material. Between the dielectric and the microwave emitter and also between the transporting belt and the metallic wall of the chamber there is an air gap or a different dielectric medium. Thus, the electrodynamic problem reduces to simulating the propagation of an electromagnetic wave in a multilayer dielectric medium.

To solve the present problem, we simplify it assuming normal incidence of the wave on the surface of the object subjected to thermal treatment [1]. Then, from the Maxwell equations for the case of harmonic vibrations, we write the following wave equation, which is valid for each layer of the dielectric medium:

$$\frac{d^2E_n}{dz^2} = k_n^2 E_n. \tag{6}$$

Here E_n is a complex function of the electric intensity in a layer with number n, z is the coordinate in the direction of propagation of the electromagnetic wave, $k_n = \alpha_n + j\beta_n$ is the propagation factor, and α_n and β_n are the damping factor and the wavenumber, which depend on the electric properties ε'_n and $\tan\delta_n$. The solution of Eq. (6) has the form

$$E_n = A_n \exp(-k_n z) + B_n \exp(k_n z).$$

The magnetic intensity in each layer is

$$H_n = \frac{k_n}{j\omega\mu_n\mu_0}\left[A_n \exp(-k_n z) - B_n \exp(k_n z)\right].$$

The integration constants A_n and B_n are determined from the following boundary conditions:

$$E_0 = A_1 + B_1 \qquad \text{for} \quad z = 0,$$

$$E_{n-1} = E_n, \quad H_{n-1} = H_n \quad (n = 2, \dots, N-1) \qquad \text{for} \quad z = l_n,$$

$$E_n = 0 \qquad \text{for} \quad z = l_N,$$

where E_0 is the electric intensity in the plane of the antenna aperture and N is the number of dielectric layers. Usually, one does not know E_0 but knows the density of the power radiated by the antenna: $p_0 = 0.5\,\mathrm{Re}\,(E_0\bar{H}_0)$, where $\bar{H}_0$ is the conjugate complex magnetic intensity at $z = 0$. This relation together with the boundary conditions are used to calculate the constants A_n and B_n and the distributions $E(z)$ and $q_v(z)$. The function $q_v(z)$ is required to solve the heat- and mass-transfer problem.

We consider the heat- and mass-transfer problem for processes of microwave drying of materials at temperatures below 373 K, at which there is no excessive steam pressure. This regime is used in the thermal treatment of materials for which occurrence of considerable internal pressures, leading to large strains or failure, is intolerable. In this case, for a fixed dielectric being treated, the heat- and mass-transfer equations (2) and (3) become

$$\frac{\partial W}{\partial \tau} = a_w \nabla^2 W + a_w \varphi \nabla^2 T, \qquad \frac{\partial T}{\partial \tau} = a\nabla^2 T + b\frac{L}{c}\frac{\partial W}{\partial \tau} + \frac{q_v}{c\rho}, \tag{7}$$

where a is the thermal diffusivity, b is the factor of internal evaporation of moisture, L is the heat of evaporation, a_w is the diffusivity of moisture, and φ is the relative thermal diffusivity. The initial temperature and the moisture content are uniform throughout the volume: $T = T_0$ and $W = W_0$ at $\tau = 0$. The boundary conditions on the outer surface of the object have the form

$$a_w \nabla W + h_w(W - W_-) = 0, \qquad \lambda \nabla T + h(T - T_-) = 0, \tag{8}$$

where h_w and h are the mass- and heat-transfer coefficients and W_- and T_- are the moisture content and the ambient temperature.

To solve the heat- and mass-transfer problem, we use the finite element method, according to which the solution of Eqs. (7) subject to boundary conditions (8) is equivalent to finding a minimum of the functionals

$$\chi_1 = \int\limits_V \Big\{ \frac{a_w [\mathrm{grad}\, W]^2}{2} - \Big[-\frac{a_w \varphi}{\lambda} q_v - \Big(1 + \frac{a_w \varphi}{\lambda} b\rho L\Big) \frac{\partial W}{\partial \tau} + \frac{a_w \varphi}{\lambda} c\rho \frac{\partial T}{\partial \tau} \Big] W \Big\} \, dV$$

$$+ \int\limits_S \frac{h_w}{2} (W - W_-)^2 \, dS; \tag{9}$$

$$\chi_2 = \int\limits_V \Big\{ \frac{\lambda [\mathrm{grad}\, T]^2}{2} - \Big[q_v - c\rho \frac{\partial T}{\partial \tau} + b\rho L \frac{\partial W}{\partial \tau} \Big] T \Big\} \, dV + \int\limits_S \frac{h}{2} (T - T_-)^2 \, dS. \tag{10}$$

Here V and S are the volume and outer surface of the body. Functional (9) allows for moisture transfer by both concentration diffusion and thermal diffusion. We assume that within each of the elements into which the region V is divided, the temperature and moisture content depend linearly on the coordinates.

Expressing functionals (9) and (10) in terms of the temperature and moisture content at nodes, performing minimization, and employing a central difference scheme [5], we obtain systems of algebraic equations for the moisture content and temperature at the nodes. We write these equations in matrix form:

$$([K_{11}] + 2d[C_{22}]/\Delta\tau)\{W\}_+$$
$$= 2d[C_{22}]\{W\}_i/\Delta\tau + (\{F_1\}_+ + a_w\varphi\{F_2\}_+/\lambda - a_w\varphi[K_{22}]\{T\}_+/\lambda); \tag{11}$$

$$([K_{22}] + 2[C_{22}]/(g\Delta\tau))\{T\}_+$$
$$= 2[C_{22}]\{T\}_i/(g\Delta\tau) + (\{F_2\}_+ + b\rho L\{F_1\}_+/g - b\rho L[K_{11}]\{W\}_+/g). \tag{12}$$

Here $\{T\}_+ = 0.5(\{T\}_{i+1} - \{T\}_i)$ and $\{W\}_+ = 0.5(\{W\}_{i+1} - \{W\}_i)$ (i is the time step number), $\{F_1\}_+$ and $\{F_2\}_+$ are the mean load vectors in time $\Delta\tau$, $\{W\}_+$ and $\{T\}_+$ are the required vectors of moisture content and temperature at the nodes for the middle of the interval $\Delta\tau$; $[K_{11}]$, $[K_{22}]$, $[C_{11}]$, and $[C_{22}]$ are the coefficients of the matrix equations obtained by minimizing functionals (9) and (10), $g = 1 + a_w\varphi b\rho L/\lambda$, and $d = (1 + a_w\varphi b\rho L/\lambda - a_w\varphi/\lambda)/(c\rho)$.

Systems (11) and (12) are solved by successive approximations. In each time step, the thermal diffusion of moisture is first ignored ($\varphi = 0$). Thus, Eq. (11) becomes independent of (12). This makes it possible to determine the moisture content at the nodes $\{W\}_+$ from (11) and then to find the temperature at the nodes $\{T\}_+$ from $\{W\}_+$ and initial temperatures $\{T\}_i$. In the next iteration, systems (11) and (12) are solved again, but in (11) the temperature at the nodes $\{T\}_+$ becomes known from the previous iteration, and this allows thermal diffusion to be taken into account. After the required accuracy of the moisture content is attained, the iterations are completed. A similar procedure is performed for the next steps $\Delta\tau$.

The temperature field obtained by simulation is used to solve the thermoelastic problem, which is considered in a quasistationary approximation. Following the finite-element method, which is also employed at the present stage of simulation, it is necessary to minimize the total potential energy in the region considered [6]:

$$\Pi = \sum_{e=1}^{m} \Big[\int\limits_{V^{(e)}} 0.5\{U\}^{\mathrm{t}}[G^{(e)}]^{\mathrm{t}}[M^{(e)}][G^{(e)}]\{U\} \, dV - \int\limits_{V^{(e)}} \{U\}^{\mathrm{t}}[G^{(e)}]^{\mathrm{t}}[M^{(e)}]\{r^{(e)}\} \, dV \Big]. \tag{13}$$

Here $V^{(e)}$ is the region occupied by an individual element, m is the number of elements, $\{U\}$ is the nodal-displacement vector, $[G^{(e)}]$ and $[M^{(e)}]$ are matrices of gradients and elastic characteristics, and $\{r^{(e)}\}$ is the strain vector, related to the heat expansion of the dielectric material.

From the minimum condition (13), we obtain a system of algebraic equations for nodal displacements, which is solved by the Gauss method. From the nodal displacements obtained, the strains and temperature stresses are evaluated using (5).

The interrelated processes of microwave thermal treatment of materials with physical properties varying with time are simulated using the following general approach. The time of thermal treatment is divided into intervals within which the physical properties can be considered constant and the difference scheme can be considered steady [6]. In each step $\Delta\tau$, the problems of electrodynamics, heat and mass transfer, and thermoelasticity are solved sequentially. From the calculation results, the physical properties of the material are established for the following step $\Delta\tau$, and the procedure is repeated up to the moment of termination of the process.

The proposed mathematical models are used to control and optimize technological microwave processes and beam-type apparatus. One primal problem of controlling processes of microwave thermal treatment is to change the object from the initial state to the desired state. Depending on the technological features, the state of the object is characterized by temperature, moisture content, temperature stresses, strains, and other parameters.

To transfer the system from the initial state $Q(M)$ to the specified state Q_+, we use the following optimality criteria [7]:

— the accuracy of transfer at the specified time $J = \max\limits_{D} |Q(M, \tau_+, q) - Q_+| \leqslant \psi$;

— the rate of transfer $J = \max\limits_{D} |Q(M, \tau, q) - Q_+| \leqslant \psi$, $\tau \to \min$.

Here M is a point that belongs to the examined region D, τ is the specified time, q is a control function, which depends on the type of control action, and ψ is the permissible departure from the specified state.

These problems are solved under limitations due to technological requirements and features of the microwave apparatus used. The main limitations for the processes of thermal treatment of dielectrics in microwave chambers of the beam type are: the maximum temperature of the object $\max T(M, \tau, q) \leqslant T_*$, where T_* is the permissible heating temperature, the largest value of the main or maximum shearing stresses (depending on the mechanical characteristics of the material) $\max \sigma(M, \tau, q) \leqslant \sigma_*$, the moisture content $W_- \leqslant W \leqslant W_+$, the ambient temperature $T_- \leqslant T_0 \leqslant T_+$, the heating rate of the material $\partial T/\partial\tau \leqslant \Theta$, the power of the microwave emitter $P_- \leqslant P \leqslant P_+$, consumption of microwave energy in the time of control $\int\limits_0^{\tau_+} P(\tau)\, d\tau \leqslant P_*$ (P_* is the permissible consumption of power), the position occupied by the center of the moving antenna $S(\tau)$ above the surface being treated F: $S(\tau) \in F$, the speed of the antenna $V_- \leqslant V(\tau) \leqslant V_+$, and the standing-wave factor, which defines the extent to which the microwave chamber matches the transfer line $1 \leqslant K \leqslant K_*$.

One important problem of control is to obtain the desired temperature field distribution by creating an appropriate distribution of internal heat sources in the volume of the body.

In the present work, we studied the possibilities of controlling internal heat sources in the microwave heating of a plate made of beech in a beam-type chamber (Fig. 1). The controlling parameter was the distance Δl between the heated object and the metallic reflecting wall.

Calculations were performed for the following initial data: plate thickness $\Delta L = 5$ cm, mean density of microwave power $p_0 = 3$ W/cm^2, frequency $f = 2450$ MHz, initial temperature of the plate $T_0 = 293$ K, $\lambda = 1.15$ W/(m$\cdot$K), $c = 1717$ J/(kg$\cdot$K), $\rho = 1560$ kg/m^3, $\varepsilon' = 3.4$, $\tan\delta = 0.17$, and ambient temperature $T_+ = 293$ K.

It was assumed that the plate length along the Oy axis was much greater than its cross section, and the power of internal heat sources was distributed uniformly in this direction. Therefore, heat transfer along

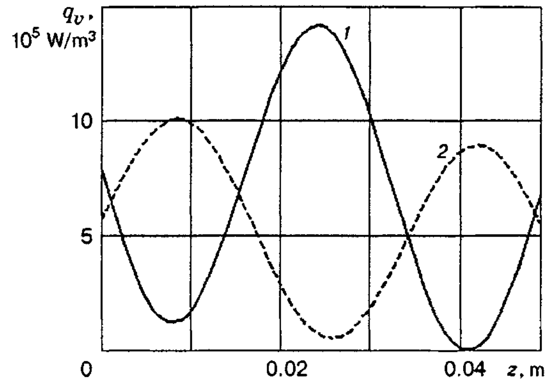

Fig. 2

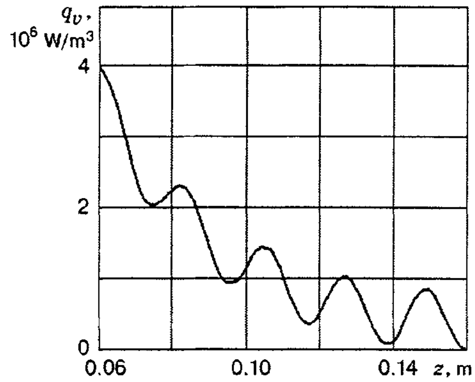

Fig. 3

Fig. 2. Distribution of the power of internal heat sources across the timber thickness for $\tau = 3$ min: $\Delta l = 5$ (1) and 1 cm (2).

Fig. 3. Distribution of the power of internal heat sources across the plate thickness for $x = 0$.

the Oy axis was ignored and the two-dimensional problem of heat conduction for the cross section of the plate was considered (Fig. 1). The thickness of the conveyer belt was small and had negligible thermal resistance. Therefore, third-order boundary conditions were specified over the entire outer surface. The heat-transfer coefficient was determined with allowance for convection and radiation.

It is established that the distribution $q_v(z)$ has the form of a standing wave for which positions of minima and maxima depend appreciably on the distance Δl between the plate and the metallic wall (Fig. 2). The temperature distribution $T(z)$ is similar in nature. At $\Delta l = 5$ cm, the maxima of $q_v(z)$ and $T(z)$ are in the middle of the plate, and the minima are at $z = 1$ cm and $z = 4$ cm. After 3-min heating, the maximum temperature is 378 K and the minimum temperatures are 303 and 312 K.

At $\Delta l = 1$ cm, the distributions $q_v(z)$ and $T(z)$ are different: the maximum temperatures are 356 K at $z = 1$ cm and 347 K at $z = 4$ cm and the minimum temperatures are 306 K at $z = 2.5$ cm. Controlling the temperature distribution by changing the distance Δl, it is possible to activate the technological processes. For example, in microwave drying, attainment of maximum temperatures in the middle of the object facilitates faster removal of moisture [3]. To obtain uniform heating over the entire volume of the object, it is possible to change the position of the metallic wall periodically, thus displacing the maximum intensity across the thickness of the material.

An important problem of thermal treatment of dielectric materials is to determine optimal regimes for microwave heating in which the temperature stresses do not exceed permissible values. This problem was solved for the case of high-temperature thermal treatment of the constructions of storehouses of harmful chemical agents with the purpose of detoxification. The thermal treatment of a flat concrete plate 10 cm thick acted upon by an immovable microwave emitter was simulated.

It was necessary to solve the following optimization problem: to determine the power of the microwave emitter, operating on a frequency of 2450 MHz, for which the maximum temperature reaches $T_* = 690$ K in time $\tau = 1$ min, and the maximum value of the main temperature stresses does not exceed 48 MPa.

Simulation was performed under following conditions. The microwave emitter had a Gaussian distribution of the power of internal heat sources with the center coinciding with the middle of the cross section of the plate. The power of the internal heat sources was distributed uniformly along the length of the plate. The distance between the emitter and the plate and the distance between the plate and the reflecting metallic wall were equal to 6 cm. Third-order boundary conditions were imposed on the outer surface of the plate.

The thermoelastic problem was solved for the case of plane deformation of the cross section of the plate under conditions of zero displacements of the middle points of the plate in the Ox direction and rigid

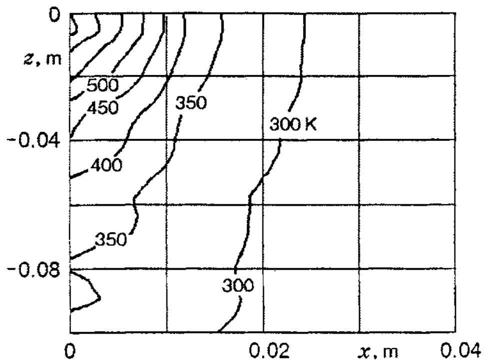

Fig. 4

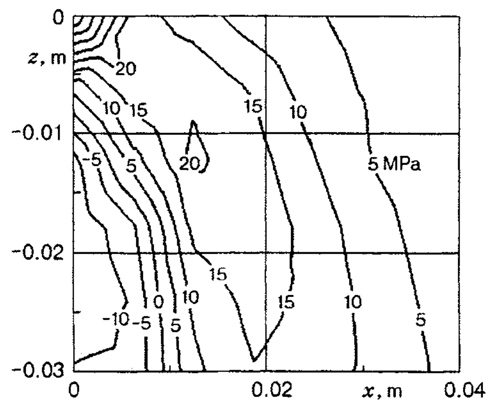

Fig. 5

Fig. 4. Isotherms in the concrete plate for time $\tau = 45$ sec.

Fig. 5. Distribution of the highest principal stresses in the upper layer of the concrete plate for time $\tau = 45$ sec.

fastening of the point A (see Fig. 1).

Simulation results are shown in Figs. 3–5. By virtue of the symmetry of the temperature and mechanical-stress fields about the middle of the cross section of the plate, Figs. 4 and 5 show the temperature and stress distributions only for half the plate. The decrease in the power across the depth of the plate is pronounced (see Fig. 3). This is explained by the fact that the thickness of the plate exceeds the depth of penetration of the electromagnetic wave into concrete ($\Delta = 6.9$ cm). Therefore, the concrete layers away from the emitter are heated only slightly.

It is established that for a time of thermal treatment $\tau = 45$ sec, the optimal power of the microwave emitter is equal to 2.5 kW. Thus, the maximum temperature has is 691 K, and the principal stresses $\sigma = 44.8$ MPa are less than $\sigma_* = 48$ MPa. Two-sided microwave heating by several emitters can be recommended to obtain a more uniform temperature field.

Thus, the proposed numerical procedure of simulating processes of electrodynamics, heat and mass transfer, and thermomechanics can be used to solve control and optimization problems as applied to the thermal treatment of dielectrics in microwave apparatus of the beam type.

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